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ON THE APPLICATION OF NONSTATIONARY ANALOGY FOR THE DETERMINATION OF HYPERSONIC FLOWS PAST BLUNT BODIES

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A class of nonstationary flows for which the nonstationary analogy may be directly used for calculating the hypersonic flow past a blunt body at some distance from its blunted part is indicated.

It is shown that the nonstationary analogy without the introduction in the flow field of entropy corrections, generally, necessitates a certain special distortion of the shock wave shape during the transition from a nonstationary to a stationary flow.

The known solutions of equations of the nonstationary motion of gas, and the similarity of the stationary and nonstationary flows, established in [1-3], are often used in calculations of hypersonic flows around blunted slender bodies. The most frequently used are the solutions for strong explosions [4] and those for a piston moving according to the power law [5]. However, with the latter method of constructing solutions the entropy at the surface of the body corresponding to the piston is infinite, and in the neighborhood of this surface the solution looses its physical meaning.

This shortcoming of the theory has been corrected in [6-10] in which the inverse problem, i.e. that of finding a stationary flow corresponding to a shock, and obtained from the initial nonstationary solution by the nonstationary analogy (by the substitution $x = u_\infty t$). The method of introducing the so-called entropy corrections to the shape of the body and to flow parameters derived directly from nonstationary analogy was proposed in those papers. It was shown in [9] that, in particular, in the case of a strong explosion such corrections reduce to a modification of only the shape of the body, whose surface must be assumed to follow the streamline corresponding to the trajectory of that particle of the nonstationary flow whose entropy equals that obtaining downstream of a normal shock. The complete class of flows having this property will be indicated in the following. The analysis of other flows in this formulation of the inverse problem is made much more difficult by the necessity of introducing corrections not only to the shape of the body but, also, to the flow field [10]. With the aim of determining a certain class of stationary flows around blunt bodies, in this paper we propose a different method of construction (of solutions) which avoids these difficulties. The underlying idea of this method is to introduce corrections to the shape of the bow shock derived by nonstationary analogy, and not to the flow field. The shape of this shock is selected on the basis of the condition of complete congruence of the fields of stationary and nonstationary solutions, while the shape of the body corresponds to the trajectory of a particle of the nonstationary flow whose entropy equals that obtaining behind the normal shock. With this construction there is no need for any corrections to the shape of the body or to the flow field.

All of the above obviously relates to flow regions in which the theory of plane flow is applicable.

Let us consider the possibility of using the nonstationary analogy for determining stationary hypersonic flows of a perfect gas past plane or axisymmetric blunt bodies, and determine the conditions under which the parameters of a known nonstationary solution, recalculated by the use of nonstationary analogy, coincide with the parameters of a certain stationary flow within the framework of the theory of plane flows.

Let the shock waves $r = R_*(x)$ and $r = R(x)$ correspond in a Cartesian (or cylindrical) coordinate system (Fig. 1) to a nonstationary and to the sought stationary flows, respectively. We introduce notation: p for pressure, ρ for density, u_∞ for the velocity of the unperturbed flow, t for time, and $\varkappa$ for the adiabatic exponent. Using the substitution $t = x / u_\infty$, we transfer the nonstationary solution to the rx-plane. Let us assume that the gas pressure in the unperturbed stream is negligibly small. The values S and S_* of a certain entropy function, proportional to $\rho^\varkappa / p$, behind a stationary and nonstationary shock, respectively, can then be written as

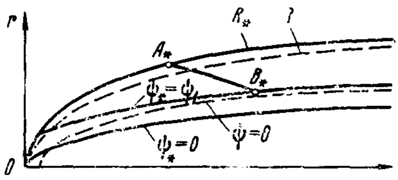

Fig. 1

$$S = \frac{C}{u_\infty^2}(1 + X'^2), \quad S_* = \frac{C}{u_\infty^2} X_*'^2, \quad X' = \left(\frac{dR}{dx}\right)^{-1} \tag{1}$$

Here C is a constant dependent on the adiabatic exponent. The entropy in the stationary flow attains its maximum behind the normal shock, i.e. at $S = S_1 = Cu_\infty^{-2}$.

Let for $X_*' = 1$ and $R_* = r_1$, $x = x_1$ in the initial flow $S_* = S_1$. Let us, also, assume that in this flow $X_*' > 1$ (or $R_*' < 1$) when $x > x_1$, since the initial flow region corresponding to streamlines which had crossed the part of the shock wave of slope $R_*' > 1$ cannot be used in the construction of stationary flow. Behind the shock the values $R_*' > 1$ are obviously possible for $x < x_1$, since in the following this part of the nonstationary solution is not used in the construction.

Let us define the stream function ψ so that at the shock wave $\psi = R^\nu$, where $\nu = 1, 2$ for the plane and axisymmetric cases, respectively. We construct the shock wave in the stationary flow so as to satisfy condition

$$S(\psi) = S_*(\psi + \psi_1) \qquad (\psi_1 = r_1^\nu) \tag{2}$$

Hence, in accordance with (1)

$$X'^2(r) + 1 = X_*'^2 [(r^\nu + r_1^\nu)^{1/\nu}] \tag{3}$$

Hence the shock wave which satisfies condition (2) is defined by the quadrature

$$X(r) = \int_0^r \sqrt{X_*'^2 [(r^\nu + r_1^\nu)^{1/\nu}] - 1}\, dr \tag{4}$$

Let us denote the local angle of inclination of the shock wave to the x-axis by τ, and consider only such stationary flows in which R_*' decreases with $x \to \infty$ in such a way that

$$R(x) = R_*(x)(1 + O(\tau^2)), \text{ or } X(r) = X_*(r)(1 + O(\tau^2)) \qquad (5)$$

It can be readily shown with the use of (4) that this condition is satisfied at least by power jumps $R_* = x^n$ for $2/(2+\nu) < n < 1$.

The theory of plane sections clearly implies that for sufficiently large x, i.e. in the region of small τ, the field of a nonstationary solution defines for $\psi > \psi_1$ a certain stationary flow. (It can be shown by the substitution into equations of stationary flow the nonstationary solution that the former are satisfied to within $\tau^{2(\varkappa-1)/\varkappa}$). We shall show that this stationary flow may be joined to the unperturbed stream via the shock (4).

In a certain downstream region bounded by any characteristic A_*B_* of the second family and the streamline ψ_1 (Fig. 1), the solution is, in fact, completely defined by the shape of the wave $R_*(x)$ and by the entropy and total enthalpy distribution along A_*B_*. In accordance with condition (2) for selecting the stationary shock and with estimate (5) at $x \to \infty$ the indicated boundary conditions are the same for both the initial and the sought solutions within the accuracy of the theory of plane sections.

It has thus been shown that a nonstationary solution of the kind defined above, when transferred to the plane of stationary solution by the simple substitution $t = x/u_\infty$, defines within the limits of similarity for $x \to \infty$ and $\psi \geqslant \psi_1$ a stationary flow around a blunt body. The shape of the body corresponds in this case to the trajectory of a particle whose entropy equals that obtaining behind the normal shock, and the shape of the shock wave (for $0 \leqslant x < \infty$) is defined by relationship (4).

We stress once again that the part of the initial solution for $0 \leqslant \psi < \psi_1$ in which the entropy may exceed the entropy in the normal shock has no physical meaning, and is not used in the transition to stationary flow.

It is interesting to note that in the case of a strong explosion ($n = 2/(2+\nu)$ by virtue of (4), $R(x) \equiv R_*(x)$ for any x. This explains the result obtained in [9] by means of constructing a stationary solution corresponding to wave $R_* \sim x^{2/(2+\nu)}$.

Let us show that this is not an isolated case, and that there exist a whole class of shock waves satisfying the "conservation condition" $X_*(r) = X(r)$. In fact, in accordance with (3), this class of shocks must satisfy equation

$$X'^2(\psi + \psi_1) = X'^2(\psi) + 1 \qquad (6)$$

Since $X'(0) = 0$, the values of X' at points $x = m\psi_1 = mr_1^\nu$ are uniquely defined by the relationship (6) and are equal m (Fig. 2). The shape of the shock between these "reference points" depends on a rather arbitrary selection of this shape along one of the segments $(m-1)\psi_1 < \psi < m\psi_1$. The straight line passing through the reference points (the dotted line in Fig. 2) relates to a strong explosion

$$(n = 2/(2+\nu))$$

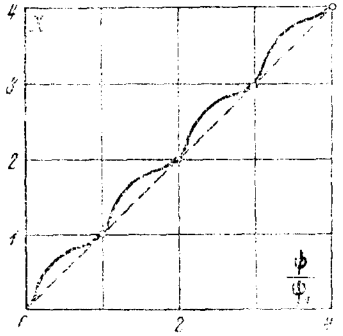

Fig. 2

For $\psi \to \infty$ all shocks waves with the "conservation property" approach the shock wave which corresponds to a strong explosion. However, when $x \to \infty$, the solutions may vary considerably throughout the flow field, since the entropy distribution at small ψ is to a considerable extent determined by the choice of the shape of the shock between the reference points.

Let us consider in somewhat greater detail the shape of bodies which would obtain as the result of application of the above method of construction to thoroughly analyzed nonstationary flows behind a shock and expanding according to the power law $R_* \sim x^n$. For $x \to \infty$ $(2 / (2 + \nu) \leqslant n < 1)$ the particle trajectories in such flows and, consequently the sought shape of the body satisfy the relationship

$$r_w^\nu = C_1 x^{\nu n} + C_2 x^{2(1-n)/\varkappa} \tag{7}$$

For $n = 2 / (2 + \nu)$ constant $C_1 = 0$ (the case of strong explosion [8]). Hence the relative value of the correction to the shape of the body (corresponding to the trajectory of the particle with $S_* = 0$) is

$$\frac{\Delta r_w}{R} = O\left(\tau^{-2/\varkappa + \nu n/(1-n)}\right) \left(n \neq \frac{1}{2}\right), \qquad \frac{\Delta r_w}{R} = O\left(\tau^{-1/\varkappa+1}\right) \qquad \left(\nu = 2,\ n = \frac{1}{2}\right)$$

The relative dislocation of streamlines consequent to the error of the theory of plane profiles is

$$\frac{\Delta r}{R} = O\left(\tau^2 + \tau^{2(\varkappa-1)/\varkappa} \frac{\Delta r_w}{R}\right)$$

Hence correction Δr_w exceeds that of the theory (Δr) when

$$\frac{1}{n} > 1 + \frac{\nu\varkappa}{2(\varkappa + 1)} \tag{8}$$

Outside this range of n the alteration of the body shape may be neglected. However the flow field in the proximity of the body $(\psi_* = \psi_1)$ differs substantially from that in the neighborhood of $\psi_* = 0$, owing to the exclusion from the flow of the region with entropy exceeding that obtaining behind the normal shock. It should be noted that according to [7, 8] in investigations of stationary shock waves of the form $R \sim x^n$ the shape of the body for $x \to \infty$ coincides with that defined by Eq. (7). However, according to [10], investigations of such flows had shown that coefficient C_2 in (7) tends to infinity when $1/n \to 1 + \nu\varkappa/(2\varkappa + 2)$. In other words, when inequality (8) is not satisfied, the shape of a body resulting in a stationary flow behind shock $R \sim x^n$ does not coincide with the particle trajectory in the corresponding nonstationary problem.

As opposed to this, the proposed here method of constructing the solution for the shape of the body at $1 > n \geqslant 2 / (2 + \nu)$ the latter is always determined by the particle trajectory in the nonstationary initial problem. The entropy of this particle is equal to the entropy obtaining behind the normal shock, and consequently the coefficient C_2 is always finite.

We note in conclusion that the questions of existence of bodies defined throughout the whole range of x by shock waves chosen in the course of solving the problem remains open, as in the works cited above.

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PARTICULAR STREAM SURFACES IN CONICAL GAS FLOWS

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Considering the field of conical flow of an ideal perfect gas near conical stream surfaces, we show that ordinary (regular) stream surfaces which are constant-entropy surfaces (isentropes) can coexist with particular stream surfaces characterized by distributed variable entropy. These particular surfaces are envelopes of the field-of-flow isentropes and can be contiguous with the regular stream surfaces without disrupting the continuity of the stream surface either in the vicinity of the particular stream surface or in the vicinity where the two surfaces meet. The results obtained enable us to postulate a pattern of nonsymmetric flow past conical bodies with a continuous and unique distribution of gasdynamic parameters in the field of flow, and to infer that this pattern is free of singular points [1].

1. Let us consider the flow of an ideal perfect gas conically symmetric with its center at the point O ; we assume that the field of flow contains a conical stream surface S on which the normal component of the flow velocity is equal to zero by definition. The stream surface S is represented by the curve S on the sphere of unit radius with its center at O (Fig. 1). We assume that in the curvilinear coordinate system η, ζ the stream surface S corresponds to $\eta = 0$ and the lines $\zeta = \text{const}$ correspond to the normals to S. In such coordinates the equations of gas motion are, for example [2], of the form

$$\begin{gathered} wu_\zeta + Xvu_\eta - X(w^2 + v^2) = 0 \\ wv_\zeta + Xvu_\eta + Xuv + Yw^2 = -\rho^{-1} Xp_\eta \\ ww_\zeta + Xvw_\eta + X_{uw} - Yvw = -\rho^{-1} p_\zeta \\ w\rho_\zeta + vX\rho_\eta + \rho(w_\zeta + Xv_\eta + 2Xu - Yv) = 0 \\ 2\varkappa(\varkappa - 1)^{-1} p + \rho(u^2 + v^2 + w^2) = \rho V^2_{\max} \end{gathered} \tag{1.1}$$